International Journal of Theoretical Physics, Vol. 37, No. 1, 1998

Continuous Measurement of Energy for a Two-Level System

Jürgen Audretsch[1] and Michael Mensky[1,2]

Received July 4, 1997

We show that it is possible to make the Rabi oscillations of the state of a two-level atom "visible" by continuous measurement of the atomic energy.

Rabi flopping is an oscillation of a two-level atom between its energy levels under the action of a monochromatic electromagnetic field (which we assume for simplicity to be resonant). It is well known that if perfect measurements of the energy of the atom (von Neumann measurements) are frequent and in the limit continuous, the interaction with the measuring apparatus prevents atomic transitions. This is the quantum Zeno effect (Misra and Sudarshan, 1977; Itano *et al.*, 1990). It is impossible in this regime of measurement to obtain information about the Rabi oscillation from the measurement readout. Moreover, the Rabi oscillation is prevented by the measurement and the readout is trivial, $E(t) \equiv E_n$.

In contrast to this, we have shown that it is possible to make the Rabi oscillation "visible" by *continuous* measurement of the atomic energy, i.e., to obtain a correlation between the measurement readout $E(t)$ and the Rabi oscillation of the state vector. This can be done with the help of continuous measurements which are fuzzy (unsharp) instead of being perfect (sharp).

Continuous quantum measurements have long been investigated (Zeh, 1971; Davies, 1976; Srinivas, 1977; Walls and Milburn, 1985), mostly with the help of models for the measuring device. A model-independent approach based on restricted path integrals (RPI) has been proposed in Mensky (1979a, b; 1998). It is equivalent to a dynamics with non-Hermitian Hamiltonians.

[1] Fakultät für Physik, Universität Konstanz, Konstanz, Germany.
[2] Permanent address: P. N. Lebedev Physical Institute, 117924 Moscow, Russia.

0020-7748/98/0100-021 5$15.00/0

An application to the measurement of energy is given in Tambini *et al.* (1995), but with the inappropriate assumption that the readout of the measurement is a constant curve $E(t) \equiv E_n$. In the following we shall correctly apply the RPI approach for systems with discrete energy spectra [see Audretsch and Mensky (1997) for details].

Instead of usual Feynman path integral, the system undergoing a continuous measurement is described Mensky (1979a, b; 1998) by a restricted path integral (partial propagator) depending on the measurement readout $[E]$. Equivalently, one may use a Schrödinger equation with a non-Hermitian effective Hamiltonian. Consider the measurement of energy H_0 in the system with the Hamiltonian $H = H_0 + V$. Then the effective Hamiltonian has the form

$$H_{[E]} = H_0 + V - i\kappa\hbar(H_0 - E(t))^2 \tag{1}$$

where κ is a parameter characterizing the measurement. If we solve the Schrödinger equation with this Hamiltonian for the initial state ψ_0, then the solution ψ_T in the final time moment describes the system state after the measurement under the condition that the measurement gave the readout $[E]$. The norm of the wave function ψ_T obtained in this way determines a probability density of the measurement output: $P[E] = \|\psi_T\|^2$.

We apply the above scheme to the case when H_0 is a multilevel system and $V = 0$. Then the probability density is found to be

$$P[E] = \exp[-(T/T_{\mathrm{lr}})\langle(E_n - E)^2\rangle_T/\Delta E^2] \tag{2}$$

where $T_{\mathrm{lr}} = 1/\kappa\Delta E^2$. This parameter is called the *level resolution time* because it determines the time sufficient for the measurement resolution to achieve the level difference ΔE. The measurement regime depends on the relation between T and T_{lr}. The two limiting regimes are following:

The level resolution regime ($T >> T_{\mathrm{lr}}$) represents a model for the decoherence process leading to distinct energy levels: (i) A readout $[E]$ is probable only if it is a constant function close to one of the levels: $E(t) \simeq E_n$; (ii) the probability that the measurement readout turns out to be close to E_n is equal to $|C_n(0)|^2$; (iii) The system is at level n after the measurement.

The level nonresolution regime ($T << T_{\mathrm{lr}}$) corresponds to the measurement being too short to distinguish between the levels: (i) Probability is high for all readouts $[E]$ having variance $E_{\mathrm{max}} - E_{\mathrm{min}}$ smaller than $\Delta E\sqrt{T_{\mathrm{lr}}/T}$; (ii) $C_n(T) \simeq C_n(0)$ for all levels between E_{min} and E_{max}; (iii) $C_n(T)$ are exponentially small for the levels separated from this band by more than $\Delta E\sqrt{T_{\mathrm{lr}}/T}$.

Let now H_0 be a two-level system and V a resonance influence of amplitude V_0. The measurement regime depends then on *three time scales*:

the measurement duration T, the level resolution time $T_{\rm lr}$, and the Rabi time $T_R = \pi/\nu$. Two limiting regimes and an intermediate one exist in this case.

The Zeno regime of measurement ($T_{lr} << T_R << T$): (i) Only readouts close to one of the levels $E(t) \simeq E_{1,2}$ have high probability; (ii) the probability for $[E]$ to be close to E_1 (or E_2) is $|\langle 1|\psi_0\rangle|^2$ (correspondingly $|\langle 2|\psi_0\rangle|^2$); iii) In the case of a readout close to E_1 (or E_2) the system is on level 1 (correspondingly level 2) after the measurement; (iv) Rabi oscillations are damped away.

The Rabi regime of measurement ($T_R << T_{\rm lr}$) with two variants:

1. If $T_R << T << T_{\rm lr}$ damping terms are negligible, then (i) Rabi oscillations are maintained, (ii) every readout $[E]$ is probable provided it lies in the band of the width $\Delta E_T = \Delta E\sqrt{T_{\rm lr}/T}$ around the levels.

2. If $T_R << T_{\rm lr} << T$, the system "forgets" initial conditions: (i) Close readouts $[E]$ lead to different final states; thus, an ensemble of readouts lying in a thin band has to be considered; (ii) contributions of different $[E]$ in this ensemble result in a mixed state not depending on the initial conditions.

The intermediate regime ($T_R \sim 2\pi T_{\rm lr} \sim T$). The measurement readout gives information about the Rabi flopping: (i) With high probability, $[E]$ lies in a relatively narrow corridor oscillating between the levels; (ii) the system state undergoes Rabi oscillations with a shifted frequency and changed shape; (iii) oscillations of $[E]$ are correlated with the (modified) Rabi oscillations.

ACKNOWLEDGMENT

This work was supported in part by the Deutsche Forschungsgemeinschaft.

REFERENCES

Audretsch, J., and Mensky, M. (1997). *Physical Review A,* **56**, 44.
Davies, E. B. (1976). *Quantum Theory of Open Systems*, Academic Press, London.
Itano, W. M., *et al.* (1990). *Physical Review A,* **41**, 2295.
Mensky, M. B. (1979a). *Physical Review D,* **20**, 384.
Mensky, M. B. (1979b). *Soviet Physics—JETP,* **50**, 667.
Mensky, M. B. (1993). *Continuous Quantum Measurements and Path Integrals*, IOP, Bristol.
Mensky, M. B. (1998). Continuously measured systems, path integrals, and information, *International Journal of Theoretical Physics*, this issue.
Misra, B., and Sudarshan, E. C. G. (1977). *Journal of Mathematical Physics,* **18**, 756.
Srinivas, M. D. (1977). *Journal of Mathematical Physics,* **18**, 2138.
Tambini, U., Presilla, C., and Onofrio, R. (1995). *Physical Review A,* **51**, 967.
Walls, D. F., and Milburn, G. J. (1985). *Physical Review A,* **31**, 2403.
Zeh, H. D. (1971). In *Enrico Fermi School of Physics IL*, Academic Press, New York, p. 263.